THE APPLICATION OF LINEAR PROGRAMMING TECHNIQUES TO AIR FORCE PROBLEMS

A NON-TECHNICAL DISCUSSION

DIRECTORATE OF
MANAGEMENT ANALYSIS
DCS/COMPTROLLER
HEADQUARTERS
U.S. AIR FORCE

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A NON-TECHNICAL DISCUSSION

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THIS PUBLICATION

IS DESIGNED TO ACQUAINT AIR FORCE PERSONNEL WITH A PHASE OF THE PROGRAMMING RESEARCH BEING CONDUCTED WITHIN THE DIRECTORATE OF MANAGEMENT ANALYSIS, DCS/COMPTROLLER.

INTERESTED PERSONNEL ARE INVITED TO CONTACT THE DIRECTORATE ABOUT THE APPLICATION OF MATHEMATICAL METHODS, SUCH AS THOSE DISCUSSED HERE, TO THEIR PROGRAMMING PROBLEMS.

foreword

1 Within the planning and programming activities of the Air Force, there are many problems which are concerned with the most efficient use of men and materiel. The solutions to these problems contribute much to the successful operation of the Air Force.

2 The development and application of methods for determining solutions to such programming problems is a continuing major research project of the Computation Division, Directorate of Management Analysis, DCS/Comptroller. With the cooperation of mathematicians, economists and other scientists, both in and out of the Air Force, mathematical procedures have been devised that solve an important class of programming problems. The study and application of these procedures form the field of Linear Programming.

3 These precise mathematical techniques are presently being employed on electronic computers by Government and civilian groups. Specifically, an electronic computer, the Univac, maintained and operated in the Pentagon by the Computation Division, has been used to solve Linear Programming problems.

the elements of linear programming problems

1 Problems in the planning and programming area are characterized by <u>limitations</u> or <u>constraints</u> imposed upon the availabilities of the items under consideration. For example, the number of men or dollars available to operate a program may have stated limitations.

2 These problems are further characterized by a <u>wide range of possible solutions</u>. That is, the constraints of such problems can be satisfied in a very large number of different ways. The selection of a particular, acceptable solution ordinarily calls for <u>judgment or experience</u>.

3

The mathematical methods of Linear Programming are designed to solve problems of this type and to rule out any subjective selection of the solution. By a systematic procedure, a single efficient solution is chosen that meets the conditions of the problem.

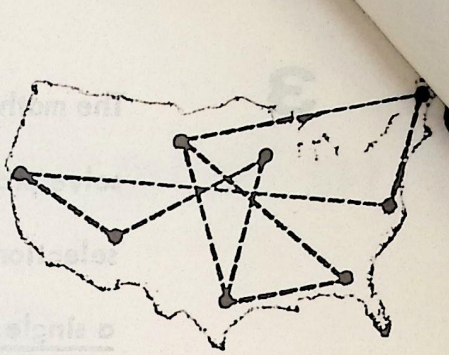
4

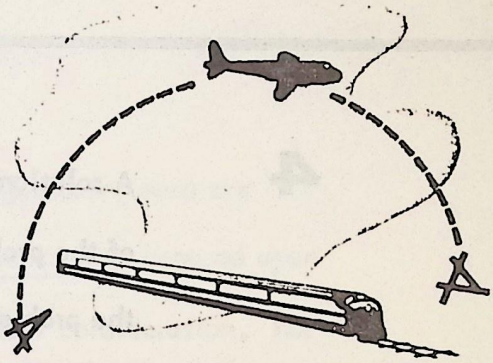
A solution is efficient if it accomplishes a specified objective of the problem. The efficient solution may, according to the problem, minimize dollar cost or expenditure of a critical item; or maximize some measure of combat effectiveness. A solution that satisfies both the constraints of the problem and the given objective is called an optimum solution.

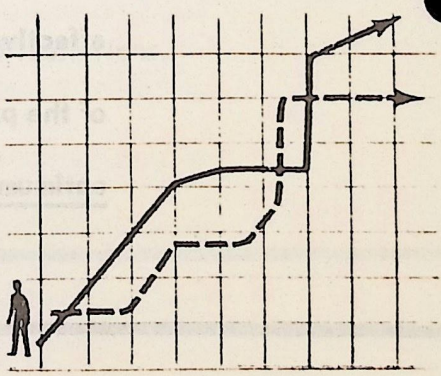
5

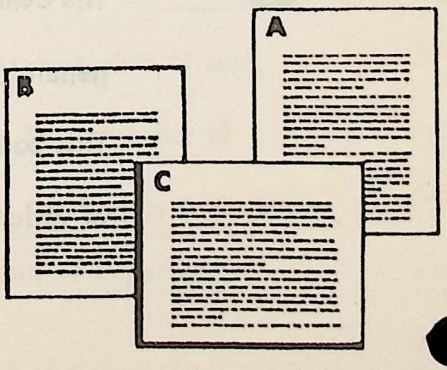
The computational methods of Linear Programming are very general. They are applicable to a large variety of problems. This booklet describes four different types of problems — all of which can be solved by the same procedures.

problems









EFFICIENT ROUTING

AIR OR SURFACE LIFT

PRODUCTION VS REQUIREMENTS

CONTRACT AWARDS

efficient routing....

THE PROBLEM

There are a number of programming problems, termed Transportation Problems, that call for the shipping of a supply of an item from a group of depots to a group of receiving stations. Each depot has a limited amount of the item, while each station has a specific requirement for the item. Usually there are many routings which will supply each station with exactly the required amount of the item. The problem is to determine the routing which not only fulfills the requirements but also minimizes some measure of the program cost. For example, the objective might be to minimize one of the following: the total dollar cost, the total number of miles of the shipping schedule, or the total time the items are in transit.

A fictitious example of a routing problem follows. Here the objective is to minimize the total ton-miles. The optimum solution in terms of this objective has been computed and is given on page 13. The reader will find it instructive if he attempts to find the most efficient routing.

EXAMPLE

Assume that Lockbourne AFB at Columbus, Ohio has been testing a large item of equipment, weighing a ton, for the B-47. It is now desired that this equipment be tried at other bases. Five each are required by March AFB at Riverside, California; Davis-Monthan AFB at Tucson, Arizona; and McConnell AFB at Wichita, Kansas. Pinecastle AFB at Orlando, Florida; and MacDill AFB at Tampa, Florida each need three. To supply these 21 items, Lockbourne AFB at Columbus, Ohio can ship 8; Oklahoma City Depot has 8; and Warner-Robins AFB at Macon, Georgia has 5. The items of equipment are to be airlifted to their destinations. The problem is to determine the routing which fulfills the requirements and minimizes the total ton-miles.

All the foregoing information together with approximate air distances are given in the following table:

	Items Available \ Required Items	Mac Dill	March	Davis—Monthan	Mc Connell	Pinecastle
		3	5	5	5	3
Oklahoma City	8	938	1030	824	136	995
Macon	5	346	1018	1416	806	296
Columbus	8	905	1795	1590	716	854

Distance In Miles

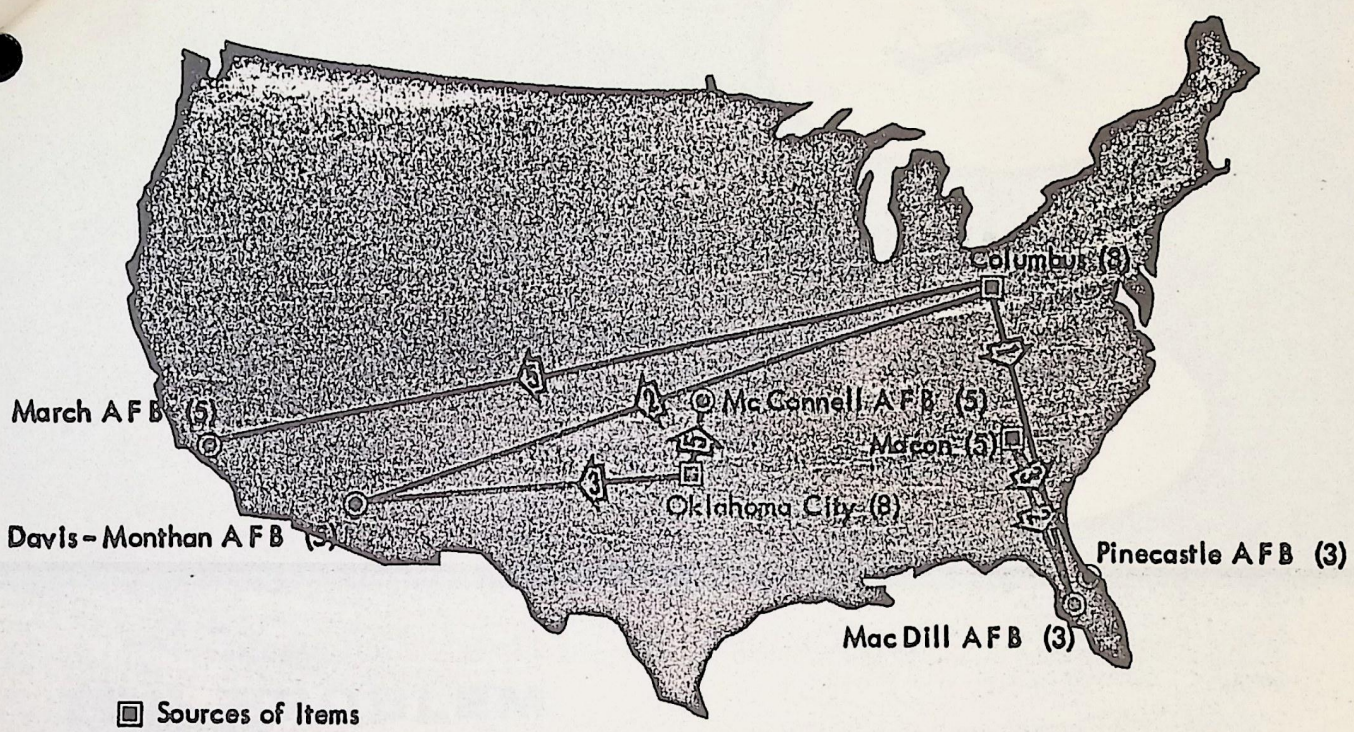
(continued) A first attempt at a solution might go like this: Try to fill each requirement from the nearest source. Let McConnell get its 5 items and Davis-Monthan 3 of its 5 items from Oklahoma City. The requirements for the Florida bases should be met, as far as possible, from Macon. Let Pinecastle get its 3 and MacDill get 2 of its 3 items from Macon. The rest of the requirements must be met from Columbus. This shipping schedule has a total of 17,792 ton-miles and is illustrated on page 13.

There are many other possible routings. The minimum solution, obtained by the methods of Linear Programming, has a value of 16,864 ton-miles. The computational procedure starts with any solution to the problem — such as the one described above — and in a systematic manner obtains better solutions until the optimum one has been determined. This Linear Programming procedure guarantees that the optimum will be found.

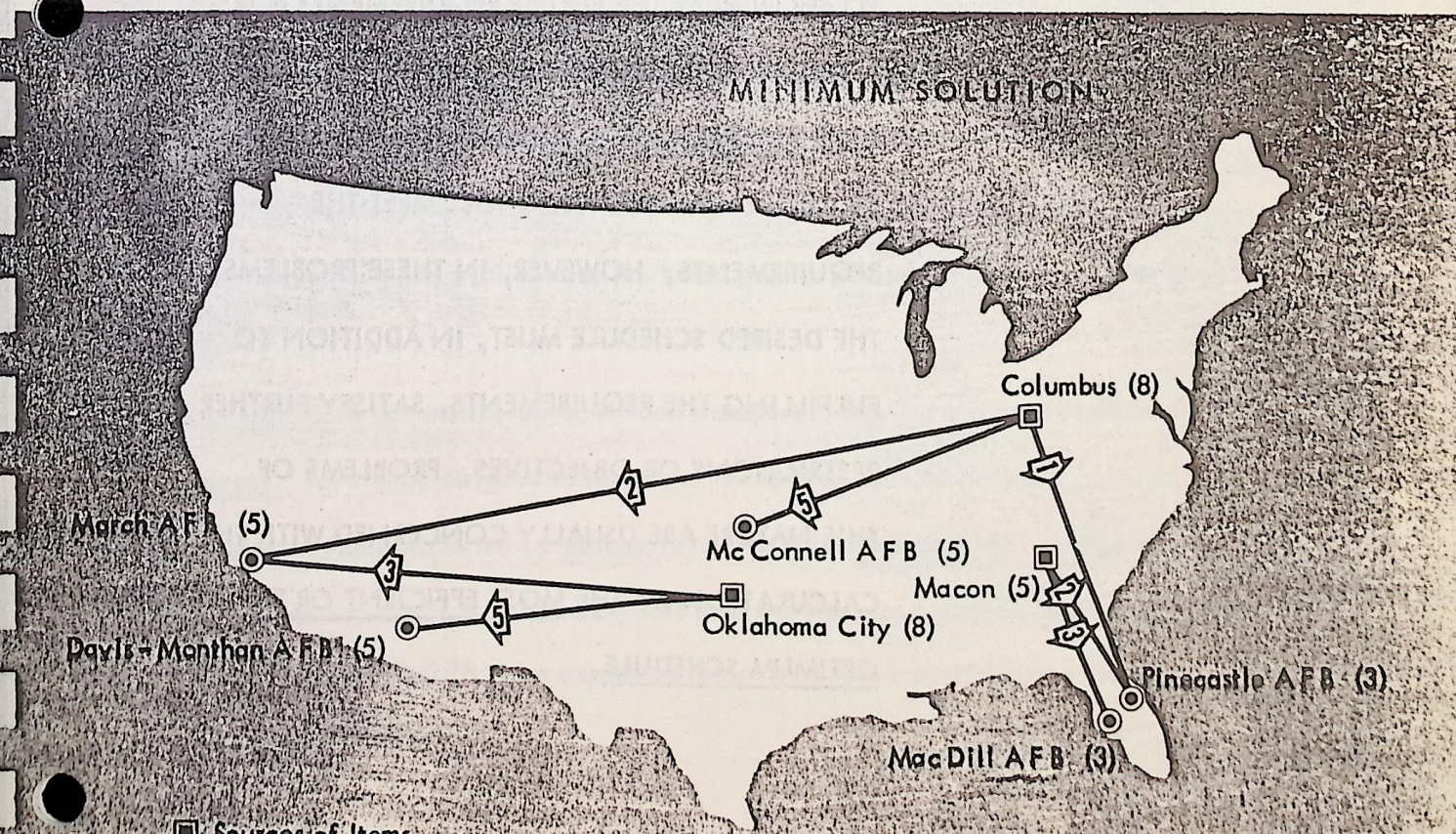
For the simple example above, it is not difficult, given the number of ton-miles in the optimum solution, to discover the corresponding routing. Without this information, one would not, in general, be able to determine just which solution is optimum. This is especially true for larger problems where solution by hand becomes quite impractical.

If the reader tried to fill some or all of Mc Connell's needs from Oklahoma City - - that is, to take advantage of the shortest distance appearing in the problem - - the resulting solution would be less efficient. This emphasizes that what at first appears to be a logical approach to problems of this nature is not necessarily the best one.

SAMPLE SOLUTION



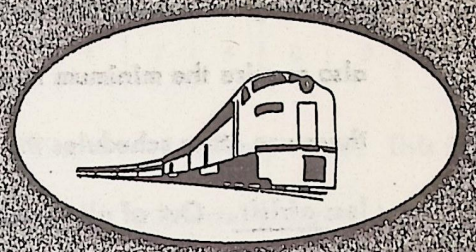
MINIMUM SOLUTION



THE NEXT TWO PROBLEMS.....

ARE CONCERNED WITH DETERMINING A SCHEDULE TO SHOW HOW THE FUTURE REQUIREMENTS FOR A SPECIFIC ITEM SHOULD BE MET. IN GENERAL, ANY SCHEDULING PROBLEM HAS MANY SCHEDULES OF THE DESIRED TYPE THAT WILL MEET THE REQUIREMENTS. HOWEVER, IN THESE PROBLEMS THE DESIRED SCHEDULE MUST, IN ADDITION TO FULFILLING THE REQUIREMENTS, SATISFY FURTHER RESTRICTIONS OR OBJECTIVES. PROBLEMS OF THIS NATURE ARE USUALLY CONCERNED WITH THE CALCULATION OF THE MOST EFFICIENT OR THE OPTIMUM SCHEDULE.

air or surface lift



THE PROBLEM

In order to accomplish its mission, a base requires monthly inputs of spare engines to meet specified engine change requirements. Engines removed for overhaul are transported from the base to an assigned repair depot. After they are overhauled, the engines are transported back to the base for possible reinstallment. If a monthly requirement for replacement engines is greater than the number of overhauled engines available at the base, new engines must be procured for that month.

The base can utilize air or surface transport in moving engines to and from the depot. An engine airlifted to the depot is airlifted back to the base. Similarly for an engine transported by surface lift. The total time an airlifted engine is in transit and in repair is, of course, less than the time for a surface lifted engine. However, since airlift is an expensive means of movement, it must be used judiciously.

(continued) The problem is to determine a schedule of monthly air and surface shipments of used engines that requires the minimum number of new engines to be procured and uses the maximum amount of surface lift. A shipping schedule that takes full advantage of the fast transport -- the one that uses airlift exclusively -- will also require the minimum number of new engines to be procured. In general, there are other schedules that call for the same number of new engines, but use less airlift. Out of all these schedules, the one that uses the least airlift is the optimum solution to the problem.

The methods of Linear Programming, in the case of the above problem, reduce to a simple hand-computational scheme for determining the optimum schedule. The results obtained in applying the scheme to a sample problem are described below. The procedure itself will not be discussed.

EXAMPLE

A base flying hour program requires a certain number of monthly engine changes, as listed in line 1 of Table I. It takes three months for an engine to be airlifted, repaired, and airlifted back to the base; and five months for a surface-lifted engine. By assuming all the engines are airlifted, the schedule given in Table I results.

TABLE I

	Months	1	2	3	4	5	6	7	8	9	10	11	12	Totals
1.	Engine Change Requirements	2	3	4	5	1	5	7	8	10	5	2	1	53
2.	Engines Airlifted From Base to Depot	2	3	4	5	1	5	7	8	10	5	2	1	53
3.	Engines Airlifted From Depot to Base	0	0	0	2	3	4	5	1	5	7	8	10	45
4.	Engines Procured to Meet Requirements	2	3	4	3	0	0	1	7	5	0	0	0	25

The total number of engines procured -- the sum of line 4 -- is twenty-five. This figure represents the minimum procurement for any schedule of air and surface shipments.

THE COMPUTATIONAL PROCEDURE IS THEN APPLIED TO THE SCHEDULES OF TABLE I. THE NUMBER OF ENGINES AIRLIFTED IS REDUCED TO A MINIMUM BY UTILIZING SURFACE LIFT WHENEVER POSSIBLE. THE TOTAL NUMBER OF ENGINES PROCURED REMAINS AT TWENTY - FIVE. THE RESULTING SHIPPING SCHEDULES AND PROCUREMENT SCHEDULE ARE GIVEN IN TABLE II. THESE SCHEDULES REPRESENT THE MOST EFFICIENT SOLUTION TO THE PROBLEM.

TABLE II

	Months	1	2	3	4	5	6	7	8	9	10	11	12	Totals
1.	Engine Change Requirements	2	3	4	5	1	5	7	8	10	5	2	1	53
2.	Engines Airlifted From Base to Depot	0	0	0	0	1	5	5	2	0	0	0	0	13
3.	Engines Surface-Lifted From Base to Depot	2	3	4	5	0	0	2	6	10	5	2	1	40
4.	Engines Airlifted From Depot to Base	0	0	0	0	0	0	0	1	5	5	2	0	13
5.	Engines Surface-Lifted From Depot to Base	0	0	0	0	0	2	3	4	5	0	0	2	16
6.	Engines Procured to Meet Requirements	2	3	4	5	1	3	4	3	0	0	0	0	25

production VS requirements

THE PROBLEM

Training schools within the Air Force are concerned with the efficient production of graduates. In order to meet a schedule of future requirements, each school must make available the required number of trained men at the right time. The manner in which the schools produce depends on the individual activity's facilities, the availability of men and materiel, and any special objectives.

Since each training activity has its own limitations and needs, an efficient method of production for one will not necessarily be efficient for another. Some training activities can readily increase their rates of output while others can do so only within limits and at substantial cost. Where, for example, equipment and personnel must be released when production drops and reinstated as it expands, large fluctuations in the level of production are undesirable. When faced with fluctuating requirements, such an activity will ordinarily overproduce in periods of low requirements, pool the surplus, and use the excess in periods of high

requirements. The production pattern can thus be made quite stable. However, such a solution may be undesirable if it yields comparatively large monthly surpluses of trained men.

Problems of this nature illustrate the difficulties that arise whenever conflicting objectives are inherent in a problem. Here the desire is to determine a schedule that minimizes both the fluctuations in output and the monthly surpluses. For these problems, efficient planning means the determination and acceptance of a middle ground solution lying between two extreme solutions -- the one that minimizes surpluses and the one that minimizes fluctuations. This optimum production schedule will depend on the relative importance assigned to the conflicting objectives.

The following example discusses a simplified version of the production problem as encountered by a technical training school. For this form of the problem, the Linear Programming procedure can generate the extreme solutions, and all production schedules that satisfy the requirements and lie between the extreme solutions.

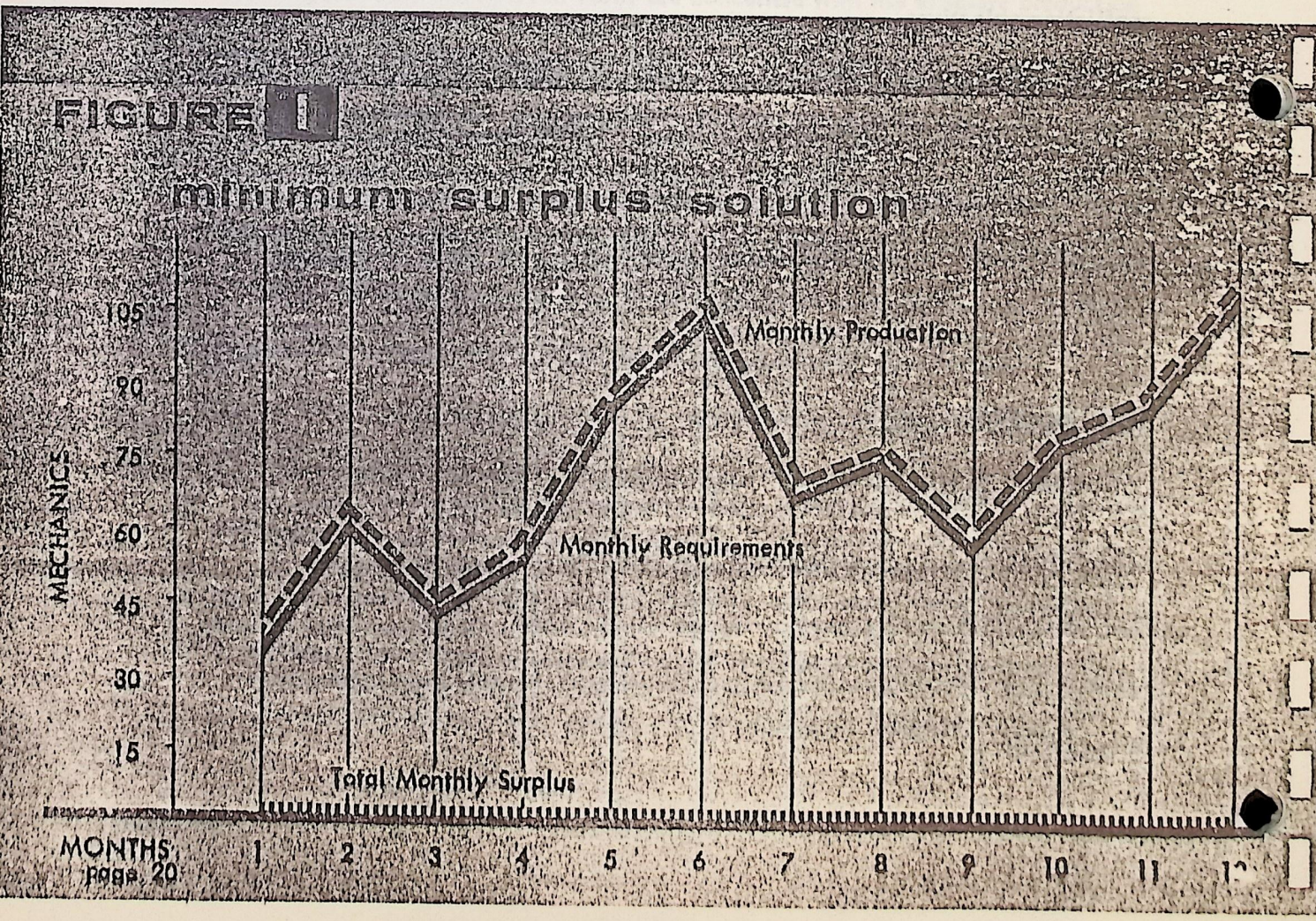
EXAMPLE

The number of monthly graduates required from an Aircraft Mechanic's Training School (for a twelve month period) is represented by the solid line in Figure 1. The total production of mechanics over the twelve month period must equal the total required, i.e. there cannot be a surplus or shortage of mechanics at the end of the program.

(continued) For this problem, the possible schedules range from the one where each man is assigned to duty on the day he graduates (minmum surplus -- Figure I) to one where a fixed training load is provided (minimum fluctuation -- Figure II). The former schedule, by producing each month the graduates as needed, requires no replacement pool, but creates the heaviest problems in operating the training school. The second schedule eliminates fluctuations in training output, but puts the greatest burden on the replacement pool. An intermediate solution (Figure III) results in a more manageable training problem which at the same time places a lighter load on the replacement pool.

FIGURE I

minimum surplus solution



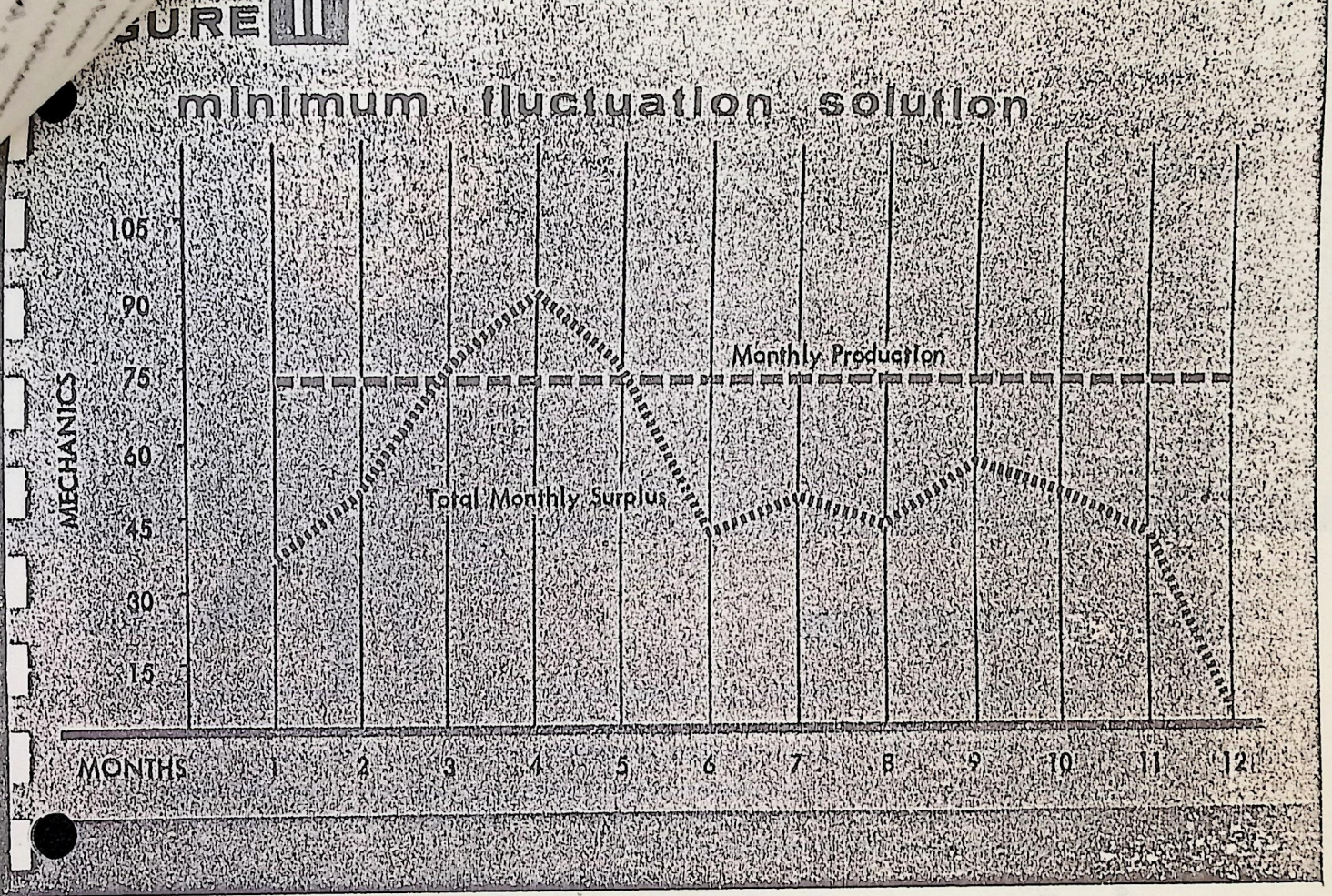
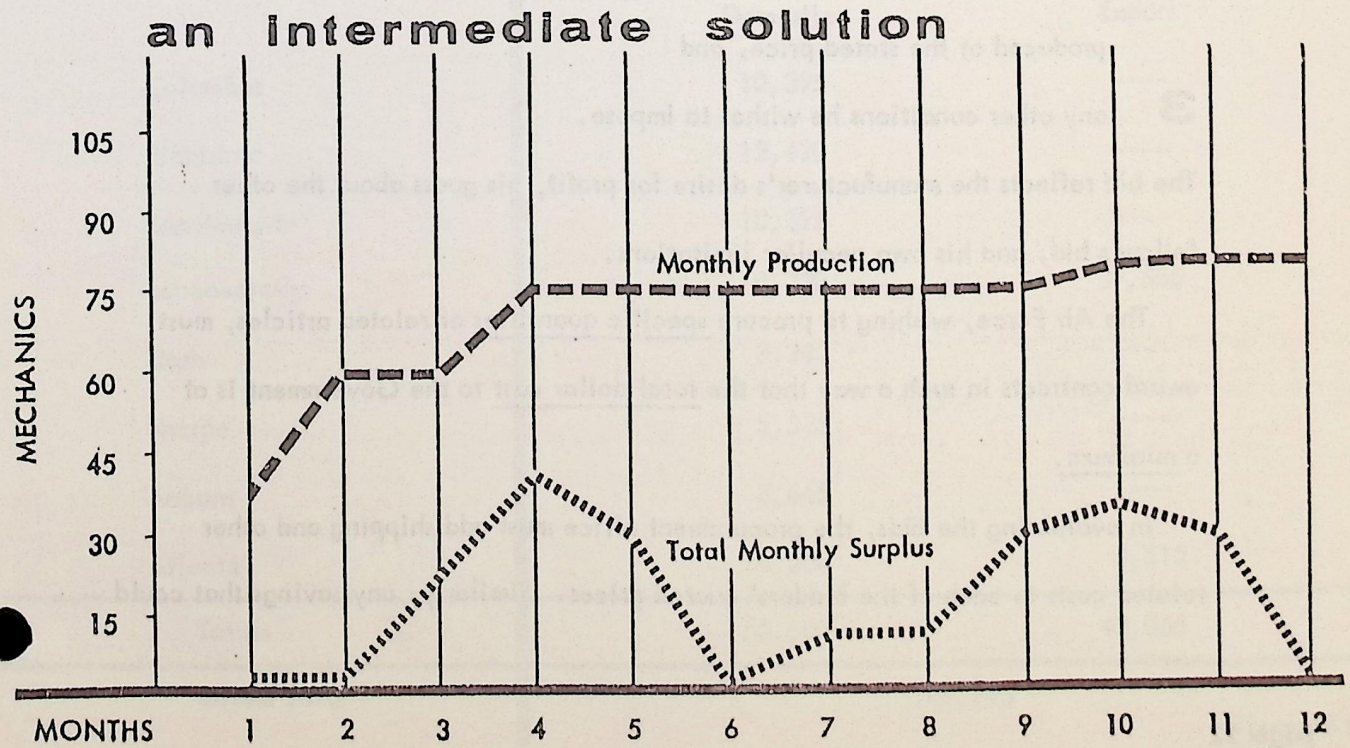

minimum fluctuation solution
105
90
75
60
45
30
15
MECHANICS
Monthly Production
Total Monthly Surplus
MONTHS
1
2
3
4
5
6
7
8
9
10
11
12

FIGURE III


an intermediate solution
105
90
75
60
45
30
15
MECHANICS
Monthly Production
Total Monthly Surplus
MONTHS
1
2
3
4
5
6
7
8
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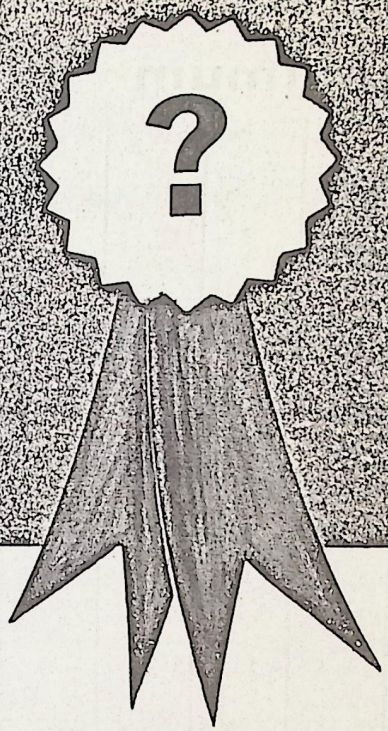
10
11
12

contract awards



THE PROBLEM

Whenever the Air Force wishes to procure items from civilian sources, producers of the items must be invited to participate in the bidding for contracts. The individual manufacturer submits bids in which he states:

1. The price per unit of article or articles,
2. the maximum (and minimum, if any) quantity of each item that can be produced at the stated price, and
3. any other conditions he wishes to impose.

The bid reflects the manufacturer's desire for profit, his guess about the other fellow's bid, and his own peculiar limitations.

The Air Force, wishing to procure specific quantities of related articles, must award contracts in such a way that the total dollar cost to the Government is at a minimum.

In evaluating the bids, the procurement office must add shipping and other related costs to each of the bidders' quoted prices. Similarly, any savings that could

be effected by agreeing to certain conditions are subtracted, e.g. a discount may be allowed for payments made within a certain time. Once the contracts have been awarded, the procurement office must be ready to demonstrate -- even to the satisfaction of the losing bidders -- that the total cost to the Air Force of all contracts was the least possible.

The following example is an actual contract awards problem as encountered by an Air Force and Army purchasing agency.

EXAMPLE

The quartermaster depots listed below require the stated amount of quires (a quire is a package of 24 sheets) of duplicating machine stencils. The amounts required are separated into "domestic" and "export" categories as the special packaging for export dictates a higher cost per unit.

	Domestic	Export
Columbus	10,395	----
Richmond	12,420	----
San Antonio	10,395	----
Schenectady	9,720	39,555
Utah	3,240	----
Sharpe	5,535	----
Auburn	3,645	----
Atlanta	10,330	3,510
Totals	65,680	43,065
Grand Total	108,745	

FOUR MANUFACTURERS SUBMIT THE FOLLOWING BIDS. THE GIVEN PRICES INCLUDE PACKAGING AND SHIPPING COSTS.

BIDDER 1

Has a minimum of 11,500 quires and maximums of 33,145 quires for domestic and 3,510 quires for export. Bid prices per quire are:

Columbus	$0.7289	Utah	$0.7247
Richmond	$0.7398	Sharpe	$0.7276
San Antonio	$0.7229	Auburn	$0.7297
Schenectady (Domestic)	$0.7406	Atlanta (Domestic)	$0.7325
Schenectady (Export)	$0.7749	Atlanta (Export)	$0.7663

BIDDER 2

Has no maximum or minimum conditions. Bid prices per quire are:

Columbus	$0.6868	Utah	$0.7358
Richmond	$0.7058	Sharpe	$0.7389
San Antonio	$0.7204	Auburn	$0.7389
Schenectady (Domestic)	$0.7075	Atlanta (Domestic)	$0.7049
Schenectady (Export)	$0.7319	Atlanta (Export)	$0.7291

BIDDER 3

Has a maximum of 60,000 quires. Bid prices per quire are:

Columbus	$0.6574	Utah	$0.6944
Richmond	$0.6489	Sharpe	$0.6973
San Antonio	$0.6904	Auburn	$0.6973
Schenectady (Domestic)	$0.6318	Atlanta (Domestic)	$0.6646
Schenectady (Export)	$0.6452	Atlanta (Export)	$0.6816

BIDDER 4

Bids only for the domestic contract and has no minimum or maximum conditions. Bid prices per quire are:

Columbus	$0.6832	Utah	$0.7306
Richmond	$0.6724	Sharpe	$0.7339
San Antonio	$0.7227	Auburn	$0.7339
Schenectady	$0.6627	Atlanta	$0.6917

The foregoing information is summarized in Table I. The solution which satisfies the conditions imposed by the bidders and minimizes the total cost to the purchasing agency is given in Table II. This award results in a total cost to the Government of $72,953.1935. The Linear Programming procedures which yield this solution guarantee that no better result is possible.

TABLE I

	Max. No. Offered / No. Required	Bidder 1 Domestic	Bidder 1 Export	Bidder 2	Bidder 3	Bidder 4
		33,145	3,510	108,745	60,000	65,680
Columbus	10,395	.7289	---	.6868	.6574	.6832
Richmond	12,420	.7398	---	.7058	.6489	.6724
San Antonio	10,395	.7229	---	.7204	.6904	.7227
Schenectady Domestic	9,720	.7406	---	.7075	.6318	.6627
Schenectady Export	39,555	---	.7749	.7319	.6452	---
Utah	3,240	.7247	---	.7358	.6944	.7306
Sharpe	5,535	.7276	---	.7389	.6973	.7339
Auburn	3,645	.7297	---	.7389	.6973	.7339
Atlanta Domestic	10,330	.7325	---	.7049	.6646	.6917
Atlanta Export	3,510	---	.7663	.7291	.6816	---

TABLE II

	Bidder 1	Bidder 2	Bidder 3	Bidder 4
Columbus	---	---	---	10,395
Richmond	---	---	---	12,420
San Antonio	---	10,395	---	---
Schenectady Domestic	---	---	4,515	5,205
Schenectady Export	---	---	39,555	---
Utah	---	---	3,240	---
Sharpe	---	---	5,535	---
Auburn	---	---	3,645	---
Atlanta Domestic	---	---	---	10,330
Atlanta Export	---	---	3,510	---

in summary...

LINEAR PROGRAMMING

IS A MATHEMATICAL METHOD WHICH CAN BE APPLIED TO MANY AIR FORCE PROBLEMS.

THE TECHNIQUES OF LINEAR PROGRAMMING

CAN BE USED TO FIND EFFICIENT SOLUTIONS TO THOSE PROBLEMS.

LINEAR PROGRAMMING

MIGHT HELP SOLVE ONE OF YOUR PROBLEMS. THE DIRECTORATE OF MANAGEMENT ANALYSIS WILL BE GLAD TO DISCUSS IT WITH YOU.

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